P.1 25.08.2020

Rules for Inequalities

$a, b, c \in \mathbb{R}$

1. $a<b \Rightarrow a+c<b+c$
2. $a<b \Rightarrow a-c<b-c$
3. $a<b$ and $c>0 \Rightarrow ac<bc$
4. $a<b$ and $c<0 \Rightarrow ac>bc$
5. $a>0 \Rightarrow \frac{1}{a}>0$
6. $0<a<b \Rightarrow \frac{1}{b}<\frac{1}{a}$

Important Sets

Natural numbers $\mathbb{N}$ $1,2,3,4\dots$

Integers $\mathbb{Z}$ $\dots -2,-1,0,1,2,3\dots$

Rational numbers $\mathbb{Q} := \frac{p}{q}$ $p\in\mathbb{Z}$ $q\in\mathbb{N}$

Real numbers $\mathbb{R}$

Irrational numbers $\mathbb{Q}' := \mathbb{R}\setminus\mathbb{Q}$

Intervals

open intervals (a,b)

closed intervals $[a,b]$

Infinity $[a,\infty)$

Absolute Values

$$|x| := \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x<0 \end{cases}$$

1. $|xy| = |x||y|$
2. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
3. $|a\pm b| \leq |a|+|b|$ Triangle Inequality
4. $|x| = D \Leftrightarrow x=-D$ or $x=D$
5. $|x|<D \Leftrightarrow -D<x<D$
6. $|x|\leq D \Leftrightarrow -D\leq x\leq D$
7. $|x-a|\leq D \Leftrightarrow a-D\leq x\leq a+D$

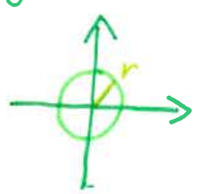
P.2

Domain & Range

$f: M \to N$

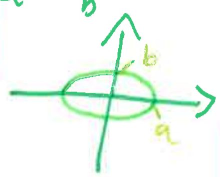
Cirkel

$y^2+x^2=r^2$



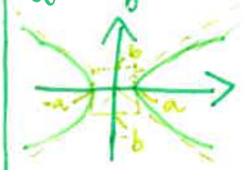
Ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=c^2$



Hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

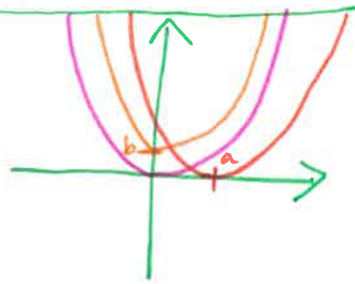


Shifting graphs

$f(x)=x^2$

$f(x)=(x-a)^2$

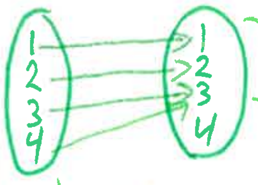
$f(x)=x^2+b$



P4

functions and graphs

$f(x): M \to N$



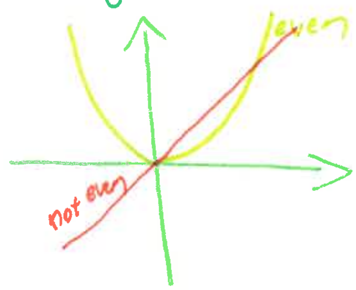
} range/image

Domain / definitions mängd

odd and even functions

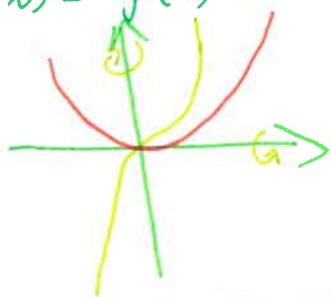
even

$f(-x) = f(x)$ y axis symmetry



odd

$f(-x) = -f(x)$ y and x axis reflection



Operation on even & odd

even + even = even
odd + odd = odd
even + odd = x
even · even = even
odd · odd = even
even ∘ even = even
odd ∘ odd = odd

P5

Composition of functions

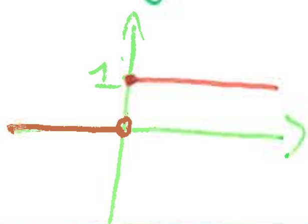
$g \circ f(x) := g(f(x))$

$\Rightarrow \quad g: A \to B$ important

$f: X \to A$

Heaviside step function

$$H(x) := \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$$



Teckenfunktionen

$\operatorname{sgn}(x) := \frac{x}{|x|}$

$$:= \begin{cases} 1 & \text{for } x > 0 \\ \text{undefined} & x = 0 \\ -1 & \text{for } x < 0 \end{cases}$$

Monotone functions

$f(x): I \subseteq \mathbb{R} \to A$

$x, y \in I \Rightarrow$

$x > y \Rightarrow f(x) \geq f(y)$ Monotonically Increasing

or

$x > y \Rightarrow f(x) \leq f(y)$ Monotonically Decreasing

Strict monotone functions

$x > y \Rightarrow f(x) > f(y)$

$x > y \Rightarrow f(x) < f(y)$ important

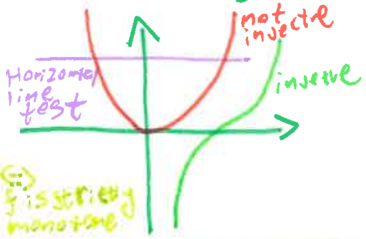
Injective functions

$f: X \to Y$

$a, b \in X$ such that

$f(a) \neq f(b)$

unless $a = b$



Surjective funtions

$f: X \to Y$

$\forall y \in Y \quad \exists x \in X$

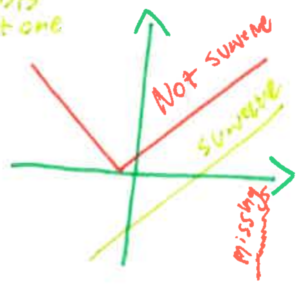
for every — there exists at least one

such that

$f(x) = y$

additionally

$img(f) = Y$



Bijective function

$f: X \to Y$

if f is surjective

AND Injective

$\Longleftrightarrow$

f is Bijective

Inverse functions

$f: A \to B$

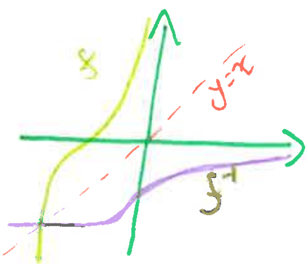
f is bijective

then there is a function

$f^{-1}(x): B \to A$

such that

$f(f^{-1}(x)) = x$



reflection along the line $y = x$

P6

Polynomials and Rational functions

deg → $\mathbb{N}$

$$P(x) := \sum_{n=0}^{N} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$$

coefficient — power — degree

alternatively

$$P(x) = c \prod_{n=0}^{N} (x - r_n) = c\underbrace{(x-r_0)(x-r_1)\dots(x-r_n)}_{n \text{ times}}$$

root — constant $c \in \mathbb{R}$ — can be complex

Rational function

$$R(x) := \frac{P(x)}{q(x)}$$

Domain of $R = \{\mathbb{R} \ni x \mid q(x) \neq 0\}$

Roots, Zeros and factors

- $P(a)$ a is a root iff $P(a) = 0$
- a is a root of P then $(x - a)$ is a factor of P
- Roots can be imagenary, ai if so they apper in pairs $(ai, -ai)$
- But real roots have real Polynomials. (not neccesarily the other way around)

Roots of a quadratic

$P(x) = Ax^2 + Bx + C = 0$

$$\Rightarrow x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Distriminant $= B^2 - 4AC$

Discriminant $> 0 \Rightarrow$ 2 real roots

$= 0 \Rightarrow$ 1 duplicate real root

$< 0 \Rightarrow$ 2 complex root pairs

There is a formula for polynomials deg 3,4 but they are too hard !!

Miscelaneus factoring

1. $ax^2 + bx = x(ax+b)$
2. $x^2 - a^2 = (x+a)(x-a)$
3. $x^3 - a^3 = (x-a)(x^2+ax+a^2)$

difference of squares
difference of cubes

4. if summ of coefficients is $=0$ then $(x-1)$ is a factor and $x=1$ a root
5. if sum of even coeffients minus sum of odd coeficents $=0$ then $(x+1)$ is a factor and $x=-1$ a root.

Polynomial Division

$$\frac{x^5+x^4+x^2+1}{x^2+1} \to$$

$$x^2 \cdot x^3 = x^5$$

$$
\begin{array}{r|l}
 & x^3 + x^2 - x \\
\hline
x^2+1 & x^5 + x^4 + x^2 + 1 \\
 & -(x^5 + x^3) \\
\hline
 & x^4 - x^3 + x^2 + 1 \\
 & -(x^4 + x^2) \\
\hline
 & -x^3 + 1 \\
 & -(-x^3 - x) \\
\hline
 & +x+1
\end{array}
$$

stop when the deg(rest) is les then the deg(denominator) i.e. $1 < 2$

$$\Rightarrow \frac{x^5+x^4+x^2+1}{x^2+1} = x^3+x^2-x+\frac{x+1}{x^2+1}$$

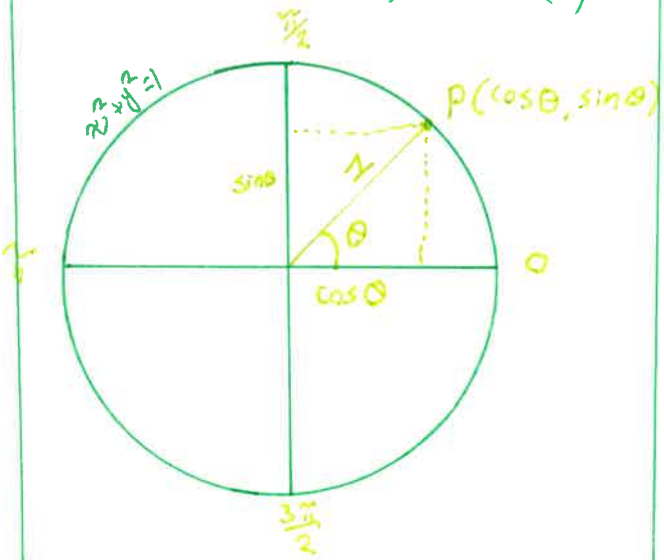
P7

Trigonometric functions

$$\sin(x) := \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(x) := \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(x) := \frac{\sin(x)}{\cos(x)}$$



arc length $s = \frac{\theta}{2\pi}(2\pi r) = r\theta$

Sector Area $A = r^2\theta$

$\Rightarrow \cos^2\theta + \sin^2\theta = 1$

Useful identities

$\cos^2\theta + \sin\theta = 1$ Pythagorean identity

$\cos(-\theta) = \cos(\theta)$ even

$\sin(-\theta) = -\sin(\theta)$ odd

$\left.\begin{array}{l}\cos(\pi-\theta) = -\cos(\theta) \\ \sin(\pi-\theta) = \sin(\theta)\end{array}\right\}$ supplementary angles $180°$

$\left.\begin{array}{l}\cos(\frac{\pi}{2}-\theta) = \sin(\theta) \\ \sin(\frac{\pi}{2}-\theta) = \cos(\theta)\end{array}\right\}$ complementary angles $90°$

Addition of angles

$\cos(\theta+x) = \cos(\theta)\cos(x) - \sin(\theta)\sin(x)$

$\sin(\theta+x) = \cos(\theta)\sin(x) + \cos(x)\sin(\theta)$

$\cos(\theta-x) = \cos(\theta)\sin(x) + \sin(\theta)\sin(x)$

$\sin(\theta-x) = \cos(\theta)\sin(x) - \sin(\theta)\cos(x)$

$\Rightarrow$

$$\left.\begin{array}{l}\sin(2\theta) = 2\sin(\theta)\cos(\theta) \\ \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) \\ \quad = 2\cos^2(\theta) - 1 \\ \quad = 1 - 2\sin^2(\theta)\end{array}\right\} \text{double-angle formulas}$$

$\Rightarrow$

$$\left.\begin{array}{l}\sin^2(\theta) = \frac{1-\cos 2\theta}{2} \\ \cos^2(\theta) = \frac{1+\cos 2\theta}{2}\end{array}\right\} \text{half angle formula}$$

Other Trig functions

$\tan(x) := \frac{\sin(x)}{\cos(x)}$

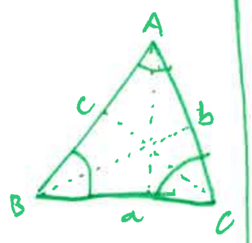
$\sec(x) := \frac{1}{\cos(x)}$

$\csc(x) := \frac{1}{\sin(x)}$

$\cot(x) := \frac{1}{\tan(x)}$

Sine law

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Cosine law

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$b^2 = a^2 + c^2 - 2ac\cos(B)$

$c^2 = b^2 + a^2 - 2ab\cos(C)$

Identities of other trig

$\sin^2(x) + \cos^2(x) = 1$

$\tan^2(x) + 1 = \sec^2(x)$

$1 + \cot^2(x) = \csc^2(x)$

3.2 Exponential functions and logaritms.

Exponentials

for $a > 0$

$a^0 = 1 \quad a^n = \underbrace{a \cdot a \cdot a \cdot a}_{n} \quad a^{-n} = \frac{1}{a^n} \quad a^{m/n} = \sqrt[n]{a^m}$

Laws of exponents

ii) $a^{x+y} = a^x \cdot a^y$ iii) $a^{x-y} = \frac{a^x}{a^y}$

v) $(a^x)^y = a^{xy}$ vi) $(ab)^x = a^x b^x$

Logarithms

log is the inverse of exponents

$y = \log_a x \iff x = a^y \quad a>0, a\neq 1$

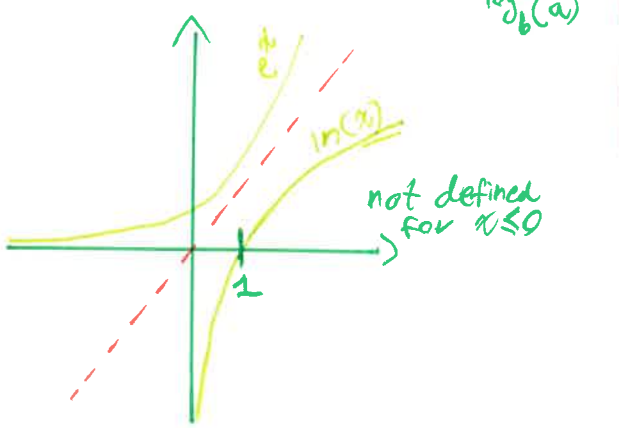
$\log_a(a^x) = x \quad a^{\log_a(x)} = x$

Laws of logarithms

i) $\log_a(1) = 0$ ii) $\log(x \cdot y) = \log(x) + \log(y)$

iii) $\log_a\left(\frac{1}{x}\right) = -\log_a(x)$ iv) $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

v) $\log_a(x^y) = y\log_a(x)$ vi) $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$



Hyperbolic functions (short) 3.6

$\cosh(x) := \frac{e^x + e^{-x}}{2}$

$\sinh(x) := \frac{e^x - e^{-x}}{2}$

$\tanh(x) := \frac{\sinh(x)}{\cosh(x)}$

$\cosh^2(x) + \sinh^2(x) = 1$

$\cosh(-x) = \cosh(x)$ - even

$-\sinh(x) = \sin(-x)$ - odd

$\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$

$\sinh(x+y) = \sinh(x)\cosh(y) + \sinh(y)\cosh(x)$

$\sinh^{-1}(x) = \ln(x + \sqrt{x^2+1})$

$\cosh^{-1}(x) = \ln(x - \sqrt{x^2-1})$

$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

Complex numbers

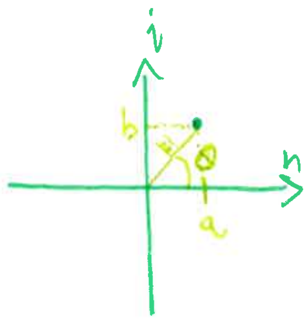
$z = a + bi \quad i^2 = -1 \quad a,b \in \mathbb{R}$

$Re(z) = a \quad Im(z) = b$

$|z| = \sqrt{z \cdot \bar{z}} = \sqrt{a^2+b^2}$

$\bar{z} = a - bi$

$\arg(z) = \theta = \tan^{-1}\left(\frac{b}{a}\right)$



operations

$z = a + bi \quad w = x + yi$

$|z||w| = |z \cdot w| \quad \left|\frac{z}{w}\right| = \frac{|z|}{|w|}$

$\arg(zw) = \arg(z) + \arg(w)$

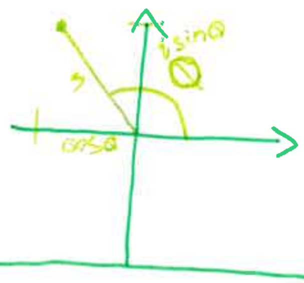
$\arg(z/w) = \arg(z) - \arg(w)$

$w \cdot z = (ax - by) + (ay + bx)i$

Eulers form

$(\cos(\theta) + i\sin(\theta)) \cdot n \quad n \in \mathbb{R}$

can represent a point in the complex plain.



Eulers formula

$e^{i\theta} = \cos(\theta) + i\sin(\theta)$

Vectors in $\mathbb{R}^2$ & $\mathbb{R}^3$

in $\mathbb{R}^2$ vector can be written as

$\vec{v} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ → x_1 and x_2 are the components

vector Addition

$\vec{v}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$\vec{v}_1 + \vec{v}_2 := \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}$

Addition works componentwise.

Scalar multiplication

$\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$ $t \in \mathbb{R}$

$t \cdot \vec{v} := \begin{bmatrix} t \cdot x \\ t \cdot y \end{bmatrix}$ t can be negative

The vector is scaled in a direction by a factor of t.

Linear combination

Sum of scalar multiples.

$\sum_{n=0}^{k} \vec{v}_n \cdot a_n$ where $\vec{v} \in \mathbb{R}^2$, $a_n \in (a_1, \dots a_n)$

$\vec{v}_0 a_0 + \vec{v}_1 a_1, \dots \vec{v}_n a_n$

zero vector

$\vec{0} := \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ the zero vector is the vector in $\mathbb{R}^n$ with only zeros as components.

$\vec{0} + \vec{v} = \vec{v}$

$\vec{v} - \vec{v} = \vec{0}$

$0 \cdot \vec{v} = \vec{0}$

Basis

A Basis of a vectorspace (eg $\mathbb{R}^2$) is a minimum set of vectors such that every vector in the vectorspace can be expressed as a linear combination of the Basis.

$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix} \right\}$ is a basis in $\mathbb{R}^2$

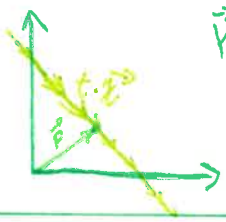
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is the standard basis

vector equation of lines

a vector equation of a line is in the form of a vector to a point in the vectorspace added to another vector that is scalar multiplied with a variable.

$\vec{Y} = \vec{P} + t\vec{x}$ $t \in \mathbb{R}$

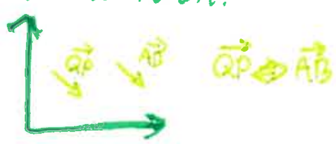
$\vec{Y} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} + t \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$



Directed line segments

on a vectorspace the vector from point Q to point P is denoted by $\vec{QP}$. Note: $\vec{QP} \neq \vec{PQ}$

two vectors are equivalent even if the positions are different.



Vectors in $\mathbb{R}^3$

the three dimentionality must follow the right-hand rule.

$\vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ vector in $\mathbb{R}^3$

$$\mathbb{R}^3 := \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \; x_1, x_2, x_3 \in \mathbb{R} \right\}$$

Linear Dependance

A set of vectors is linearely dependent iff:

There is at least one vector in the set that can be made with linear combinations of the other vectors in the set.

If not then it is linearely independent.

Length

The length between two points is the length of the vector between the two. Which is defined as

$$\left\| \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \right\| := \sqrt{x_1^2 + x_2^2 \cdots x_n^2}$$

symbol for length

or between points as:

$A = \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{matrix}$ $\quad B = (b_1, b_2 \ldots b_n)$

$$\vec{AB} = \begin{bmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_n - a_n \end{bmatrix}$$

$$|\vec{AB}| = \sqrt{(b_1 - a_1)^2 \cdots (b_n - a_n)^2}$$

Subspaces

a subspace is a nonempty set of vectors in $\mathbb{R}^n$. Where all vectors in the subspace follow the rules:

$\vec{x}, \vec{y} \in S$

1) $\vec{x} + \vec{y} \in S$ closed under addition
2) $t\vec{x} \in S$ closed under scaler multiplication

$\Rightarrow$ since t can be zero, every subspace must contain the $\vec{0}$ zero vector.

$\{\vec{0}\}$ is the trivial subspace.

Spanning set

A spanning set of a subspace is any set of vectors that is needed to be able to create a subspace through vector combinations.

Dot product (scalar product)

The dot product of two vectors is a scalar. It can be calculated in two different ways.

1) $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} := x_1 \cdot y_1 + x_2 \cdot y_2 + \cdots + x_n \cdot y_n$

2) $\vec{x} \cdot \vec{y} := |\vec{x}| \cdot |\vec{y}| \cos \theta$ Angle between the two vectors

2 can be rearranged as

$$\theta = \cos^{-1}\left(\frac{\vec{x} \cdot \vec{y}}{|\vec{x}||\vec{y}|} \right)$$

from the figure on the right it is evident that the dot product of two orthogonal vectors is equal to zero.

$\theta = 90^\circ \Leftrightarrow$ Dot product $= 0$

Cross product (vector product)

The cross product of two vectors is another vector that is orthogonal to both vectors. (Note $\vec{v}\times\vec{u} \neq \vec{u}\times\vec{v}$)

Cross product is defined in $\mathbb{R}^3$

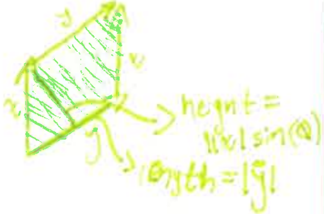
$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad \vec{x}\times\vec{y} := \begin{bmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - x_2 y_1 \end{bmatrix}$$

Length of cross product

The length of the cross product can be defined as:

$|\vec{x}\times\vec{y}| = |\vec{x}||\vec{y}|\sin(\theta)$ ← vinkeln mellan $\vec{x}$ och $\vec{y}$

This is also the Area of the parallellogram created by the vectors $\vec{x}, \vec{y}$



Lines in vectorform

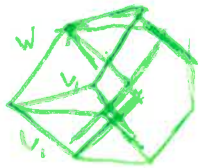
A line in $\mathbb{R}^n$ can be expressed as a point and a vektor that is scalar multiplied.

$$\vec{x} = \vec{p} + t\vec{d} \quad t \in \mathbb{R} \iff \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} + t\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix}$$

$\vec{p}$: vector to point; $\vec{d}$: direction vector.

Volume of Parallelepiped

Defined as $|\vec{w}\cdot(\vec{u}\times\vec{v})|$ where $\vec{w}, \vec{u}, \vec{v}$ are:



It's meant to look like a skew cube. Just google it LoL

Parametric line equation

is the vectorform written out for each component of the vector.

$$\underbrace{\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix} + t\begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix}}_{\text{Vector form}} \iff \underbrace{\begin{cases} x_1 = p_1 + td_1 \\ \vdots \\ x_n = p_n + td_n \end{cases}}_{\text{Parameter form}}$$

Parameter to scalarform ($y = mx + b$)

only works in $\mathbb{R}^2$.

take parameterform and eliminate t.

$x_1 = p_1 + td_1 \qquad x_2 = p_2 + td_2$

$\iff \frac{x_1 - p_1}{d_1} = t = \frac{x_2 - p_2}{d_2}$

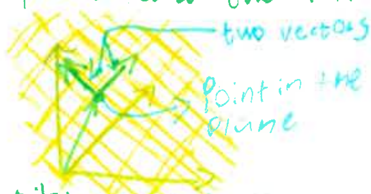
$\Rightarrow x_2 = p_2 + \frac{d_2}{d_1}(x_1 - p_1)$

Planes

Planes can be described in many ways in $\mathbb{R}^3$. One of these ways is the vector equation which consists of a point and two linearly independent vectors:

Vector form:

$$\vec{x} = \vec{p} + t_1\vec{v}_1 + t_2\vec{v}_2 \iff \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + t_1\begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \end{bmatrix} + t_2\begin{bmatrix} v_{21} \\ v_{22} \\ v_{23} \end{bmatrix}$$



Normal form: is the second way of describing a plane. using a point and a normal (orthogonal) vector to the plane, which can be obtained from the vectors $\vec{v}_1$ and $\vec{v}_2$ by using the cross product.

Normal form:

$$0 = \vec{n}\cdot\vec{px} \iff 0 = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}\cdot\begin{bmatrix} x_1 - p_1 \\ x_2 - p_2 \\ x_3 - p_3 \end{bmatrix}$$

$\vec{n}$: Normal vector, not $\vec{0}$; p: point in plane; $\vec{px}$ ← All points in plane

Scalar form is sometimes used interchangably with normal form, and it is the computed form of the normal form.

Scalar form

$$0 = n_1(x_1 - p_1) + n_2(x_2 - p_2) + n_3(x_3 - p_3) \iff n_1x_1 + n_2x_2 + n_3x_3 = d = \vec{n}\cdot\vec{p}$$

Determinants $\mathbb{R}^2$

for $\mathbb{R}^2$ the determinant is defined on a 2x2 matrix as:

$$\det(M) = \det\begin{bmatrix} A & B \\ C & D \end{bmatrix} = ad - bc$$

this is also the area of the parallelogram created by the vectors $\vec{v}_1 = \begin{bmatrix} A \\ B \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} C \\ D \end{bmatrix}$.

This means if $\det(M) = 0 \Rightarrow$ the vectors $\vec{v}_1\ \vec{v}_2$ create a parallellogram of area 0 which only occurs if $\vec{v}_1$ and $\vec{v}_2$ are either $\vec{0}$ or parallell. meaning $\vec{v}_1$ and $\vec{v}_2$ are linearly dependant.

Determinants $\mathbb{R}^3$

In $\mathbb{R}^3$ the determinant of a 3x3 matrix is defined as:

don't forget

$$\det(M) = \det\left(\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}\right) = A\cdot\det\left(\begin{bmatrix} E & F \\ H & I \end{bmatrix}\right) - B\det\left(\begin{bmatrix} D & F \\ G & I \end{bmatrix}\right) + C\det\left(\begin{bmatrix} D & E \\ G & H \end{bmatrix}\right)$$

Just like the determinant in $\mathbb{R}^2$ defines the area of a Parallellogram, in $\mathbb{R}^3$ the determenent is the volume of the Parallelpiped created by the vectors $\vec{v}_1 = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} D \\ E \\ F \end{bmatrix}$ $\vec{v}_3 = \begin{bmatrix} G \\ H \\ I \end{bmatrix}$.

Additionally if the $\det(M) \neq 0$ the vectors are lineary independent.